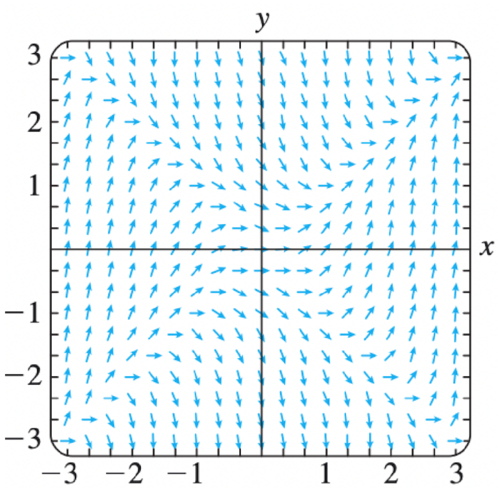
2.1 - Solution Curves Without a Solution

Definition: If $y = y(x)$ is a differentiable function, then $\dfrac{dy}{dx}$ gives the slope of a tangent line at a point. We can learn about a DE $\dfrac{dy}{dx} = f(x, y)$ using segments of tangent lines, called **lineal elements**. The collection of these lineal elements is a **direction field** or **slope field**.

Example: Sketch an approximate solution curve that passes through each of the indicated points

$$\frac{dy}{dx} = x^2 - y^2$$

a) $y(-2) = 1$ b) $y(3) = 0$ c) $y(0) = 2$ d) $y(0) = 0$



Example: Use computer software to obtain a direction field for the differential equation. Sketch an approximate solution curve passing through each of the given points.

$$y' = x + y$$

a) $y(-2) = 2$ b) $y(1) = -3$

Definition: A DE in which the independent variable does not appear explicitly is an **autonomous** differential equation. A first-order autonomous DE has the form $\frac{dy}{dx} = f(y)$.

Definition: A number c is a **critical point** of an autonomous DE if $f(c) = 0$. A critical point is also known as an **equilibrium point** or **stationary point**. A constant solution $y(x) = c$ of an autonomous DE is an **equilibrium solution**.

Example: Consider the autonomous first-order differential equation $\frac{dy}{dx} = y^2 - y^4$ and the initial condition $y(0) = y_0$. Sketch the graph of a typical solution $y(x)$ when y_0 has the given values.

a) $y_0 > 1$ b) $0 < y_0 < 1$ c) $-1 < y_0 < 0$ d) $y_0 < -1$